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Magnetic-Static Analysis of the Current Contour and Superconductor Reflector

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Abstract

The present paper deals with the exact mathematical analysis of the space distribution of a magnetic field of the current contour. Interaction of the circular shape current contour with the ideal diamagnetic / superconductor spheroid is studied. It is shown that based on the Meissner's effect the superconductor should cause the definite transformations in the space distribution of the magnetic field emitted by the current contour. Calculations being carried out represent the changes in the magnetic field arising due to application of the ideal diamagnetic reflector – the superconductor. In this case, the equal-potential line of an elongated shape is received, that sufficiently reduces the size of an emitter. Arranging several magnetic field emitters close to each other and locating high-sensitive magnetic sensors behind them will sufficiently improve the functional and technical characteristics of the devices created on the base of such construction.

Keywords: Equal-potential line, magnetic field, Meissner's effect, reflector, space distribution, superconductor.

Introduction

Magnetic field emitters are widely used in different branches of modern techniques. Change in the shape of an equal-potential line in existing magnetic field emitters usually takes place in the way of change in the geometric shape of the magnetic field current contour or by the application of ferromagnetic materials of the different shape. In such cases, the equal-potential line is the part of elliptical arc, its radius is exceeding the diameter of the current contour, while the covering area of the magnetic field is less than the size of the current contour (Kevanishvili, Mushkudiani. et al., 2013).

The aim of our study is reception of the magnetic emitter possessing the equal-potential line of elongated shape that will increase the action of the magnetic field in one preliminarily selected direction.

Statement of the problem and its solution

In recent years, improvement in properties of superconductor materials and cryogen devices arise the possibility of their utilization in an innovative way. In different areas of techniques, the problems of radiation of a magnetic field in one direction and at far distance are stated. As one of different variants of such investigations, the arrangement of the diamagnetic reflector behind the circular current coil is supposed and the superconductor is being assumed an ideal diamagnetic.

Due to it, consider the magnetic field emitter, presented in figure 1, representing the cross-section of the circular shape current contour of r radius, placed inside the superconductor spheroid of R radius. Here the diameter of the current conductor and the thickness of the superconductor

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both are ignored, as they can not sufficiently affect further calculations.

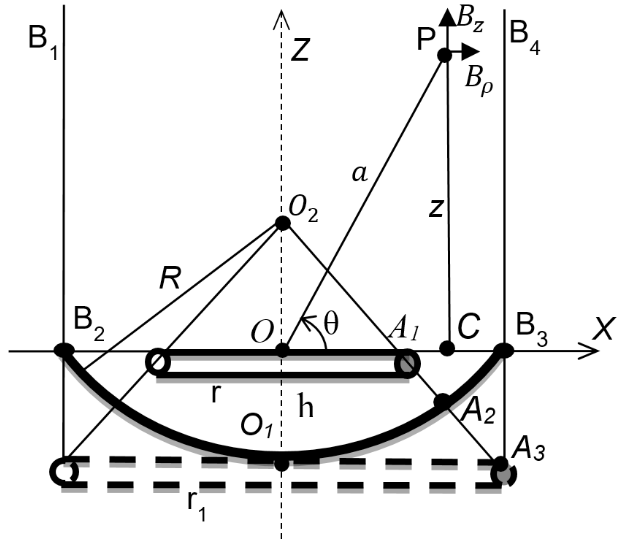

Figure 1. *Current contour inside the superconductor spheroid.*

Based on Meissner's effect, the superconductor changes the space distribution of the magnetic field emitted by the current contour. The current I passing through the contour will create the mirror image of this current at the opposite side of the superconductor in the form of the imaginary contour (Binns, & Lawrenson. 1963, p.69), which satisfies the condi¬tion $|A_1A_2|=|A_2A_3|$. It follows that the radius of this imagi¬nary contour will be given in the form of

(1) $$r_1 = 2Rr / \sqrt{r^2 + (\mathrm{R}-\mathrm{H})^2} - r$$

where $h=|OO_1|=|B_3A_3|$ is the distance between the center of the ring and the surface of the spheroid,. $\rho=|OC|$ and $z=|PC|$ are the coordinates of P point, while O_2 the center of the spheroid.

The mirror image of the contour created by the reflector will completely appear within the space surrounded by B_1B_2, B_3B_4 and B_3B_4 sides, while the magnetic field created by it will be fully mapped only into this section of space, respectively. We are now interested in the case when B_1B_2 and B_3B_4 sides are inter-parallel composing the right angle with XOY plane. Besides that, in order to decrease the non-uniformity of the surface currents arising in the superconductor, it is necessary to have $|A_1B_3|=|A_1A_2|$. For such cases it may rather easily be calculated that

$$r_1=R-r, \quad h=\sqrt{3}\,(R-2r).$$

The components of the magnetic field induction vector at P point of observation located in the positive half of *XOZ* plane, which are created without the reflector (i.e. in close direction for XOZ plane), will be (Smythe,1950, p.270]:

$$B'_\rho = \frac{\mu I}{2\pi}\frac{z}{\rho\sqrt{(r+\rho)^2+z^2}}\left[-K + \frac{r^2+\rho^2+z^2}{(r-\rho)^2+z^2}E\right]$$

$$B'_z = \frac{\mu I}{2\pi}\frac{1}{\sqrt{(r+\rho)^2+z^2}}\left[K + \frac{r^2-\rho^2-z^2}{(r-\rho)^2+z^2}E\right]$$

Where K and E are full elliptical integrals of the first and second order (Dwight, 1961. formulae 773.1 and 774.1):

$$K = \frac{\pi}{2}\left(1 + \frac{1^2}{2^2}k^2 + \frac{1^2\cdot 3^2}{2^2\cdot 4^2}k^4 + \frac{1^2\cdot 3^2\cdot 5^2}{2^2\cdot 4^2\cdot 6^2}k^6 + \cdots\right)$$
$$= \frac{\pi}{2} + \frac{\pi}{2}\sum_{n=1}^{\infty}\left(k^{2n}\prod_{m=1}^{n}\frac{(2m-1)^2}{4m^2}\right)$$

$$E = \frac{\pi}{2}\left(1 - \frac{1}{2^2}k^2 - \frac{1^2\cdot 3}{2^2\cdot 4^2}k^4 - \frac{1^2\cdot 3^2\cdot 5}{2^2\cdot 4^2\cdot 6^2}k^6 - \cdots\right) =$$
$$= \frac{\pi}{2} - \frac{\pi}{2}\sum_{n=1}^{\infty}\left(k^{2n}\prod_{m=1}^{n}\frac{(2m-3)(2m-1)}{4m^2}\right)$$

k is the modulus of the elliptical function (Smythe,1950. formula (7.50)]:

$$k = \sqrt{\frac{4r\rho}{(r+\rho)^2+z^2}}$$

ρ and *z* are the cylinder coordinates of P point of observation where $z > 0$ and $\rho > r$.

With $\rho < |OB_3|$ coordinates the components of magnetic field induction vector created by the mirror image will be given as follows:

$$B''_\rho = \frac{\mu I}{2\pi}\frac{z+h}{\rho\sqrt{(r_1+\rho)^2+(z+h)^2}}\times$$
$$\times\left[-K' + \frac{{r_1}^2+\rho^2+(z+h)^2}{(r_1-\rho)^2+(z+h)^2}E'\right],$$
$$B''_z = \frac{\mu I}{2\pi}\frac{1}{\rho\sqrt{(r_1+\rho)^2+(z+h)^2}}\times$$
$$\times\left[K' + \frac{{r_1}^2-\rho^2-(z+h)^2}{(r_1-\rho)^2+(z+h)^2}E'\right],$$

here *K'* and *E'* are full elliptical integrals of the first and second order:

$$K' = \frac{\pi}{2}\left(1 + \frac{1^2}{2^2}k_1^2 + \frac{1^2\cdot 3^2}{2^2\cdot 4^2}k_1^4 + \frac{1^2\cdot 3^2\cdot 5^2}{2^2\cdot 4^2\cdot 6^2}k_1^6 + \cdots\right) =$$
$$= \frac{\pi}{2} + \frac{\pi}{2}\sum_{n=1}^{\infty}\left(k_1^{2n}\prod_{m=1}^{n}\frac{(2m-1)^2}{4m^2}\right),$$
$$E' = \frac{\pi}{2}\left(1 - \frac{1}{2^2}k_1^2 - \frac{1^2\cdot 3}{2^2\cdot 4^2}k_1^4 - \frac{1^2\cdot 3^2\cdot 5}{2^2\cdot 4^2\cdot 6^2}k_1^6 - \cdots\right) =$$
$$= \frac{\pi}{2} - \frac{\pi}{2}\sum_{n=1}^{\infty}\left(k_1^{2n}\prod_{m=1}^{n}\frac{(2m-3)(2m-1)}{4m^2}\right),$$

where

$$k_1 = \sqrt{\frac{4r_1\rho}{(r_1+\rho)^2+(z+h)^2}}$$

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The components of the summary induction vector will be:

$B_\rho = B_\rho' + B_\rho''$ and $B_z = B_z' + B_z''$.

And the modulus of the magnetic induction will be as follows:

$$B = \sqrt{B_\rho^2 + B_z^2}\,.$$

Replacing ρ and z by a, θ we receive the dependence of the magnetic induction on the polar coordinates

$$\rho = a\cos\theta,\quad z = |\,a\sin\theta\,|$$

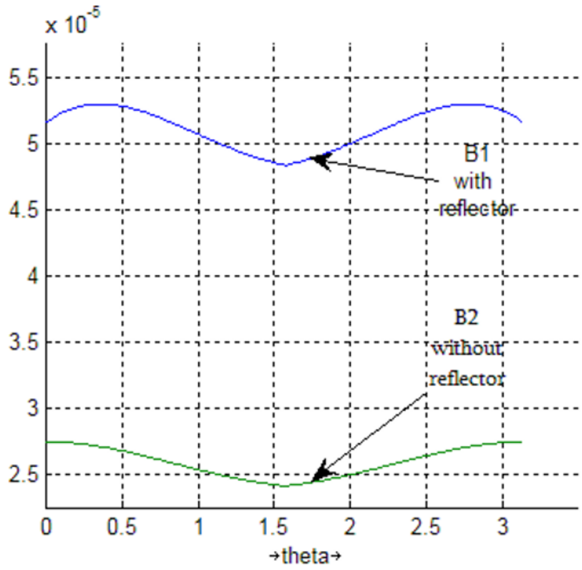

Figure 2. *Induction of the emitted magnetic field.*

Figure 2 represents the graphical performance of the magnetic field induction without the reflector B2 and with B1. The graphs are built up for the cases when r = 10 cm, R = 30 cm, α = 50 cm and I = 100 A. Calculations show that:

$$B2^{90^0}/_{B1^{90^0}} = 2.$$

Thus, the magnetic field induction of the current contour, in case of application of the superconductor reflector, in the direction normal to its plane, is twice more than when the reflector is not used.

Conclusion

Investigations being carried out arrived us to the following effects:

Application of the cryogen techniques makes possible the use of the superconductor current contour together with the superconductor reflector that, following the growth in the magnetic field strength, will sufficiently decrease the size of the emitter.

It becomes possible to arrange several magnetic field emitters and receivers side by side while their radiation patterns will superpose only within the zone of the object to be irradiated.

The emitter will sufficiently decrease the magnetic field induction in the back side of the area, and there appears the possibility of arranging the high sensitive magnetic sensor.

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