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Fourth-Order Equation Triplex Research with Root Locus Methods

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Abstract

Root locus was first used by American scientist Walter R. Evans to solve problems of analysis and synthesis of automatic control systems. W. R. Evans used Graph-analytical method to construct root locus. The Russian scientists Tedorchik and his student Bendrikov also contributed to this study.

The article is the research of Triplex polynomials with one parameter using Root Locus (RL). Roots movement trajectories analytic research revealed the roots' features that can not be detected graphically; In the article Triplex polynomials Root Locus is built.

Keywords: Charts, Polinoms, Root Locus, Trigonometric Form.

Introduction

Polynomial roots and research is essential to determine the properties of a number of technical problems to solve, in particular, the automatic control system stability and synthesis tasks. Root locus was first used by American scientist W. R. Evans. This happened in the late 40's. He also invented a special ruler "spyrul", for the construction of algebric equations roots' trajectories by which finding and summing the arguments was simplified graphically. In this Article we overview RL structuring and research of trinomial equation of Fourth order RL while coefficient are changing.

Methods

Root locus (RL) (gr. hodo - Road, Chart), is the unity of the algebraic equation roots' trajectory when the equation's coefficients, one or more are changing.

Let's start with the equation

(1) $S^4+\alpha S+1=0$

The root locus (RL) equation will be:

(2) $r^3 Sin3\varphi=Sin\varphi$

will be split into two equations:

(3) $Sin\varphi=0$

and

(4) $r^4(4Cos^2\varphi-1)=1.$

The RL starting point will be: $S^4+1=0$ equation roots: $S_1=e(j\pi/4)$; $S_2=e(j^3\pi/4)$ $S_3=e(j^5\pi/4)$ and $S_4=e(j^7\pi/4)$, which are depicted on the unit circle with the crosses (X) „Fig 1."

The RL double points will be: $S=\pm\frac{1}{\sqrt[4]{3}}=\pm0{,}76.$ Such roots are obtained, if $\propto=\mp\frac{4}{\frac{1}{\sqrt[3]{3}}}=\mp4\sqrt[4]{3}==\mp1{,}755.$

The RL areas will be:

$$\begin{cases}\sin3\varphi>0\\ \sin\varphi>0\end{cases} \text{ an } \begin{cases}\sin3\varphi<0\\ \sin\varphi<0\end{cases}$$

union inequalities system solutions:

$\varphi\in]0;60^0[\cup]120^0;180^0[\cup]-60^0;0[\cup]-120^0;-180^0[.$

RL's chart depicted in "Fig 1", which indicates the two dots and α-value of this point. In "Fig 1" and in the previous drawings, when α changes from 0 to + ∞, roots movement in the direction of the trajectory is marked with the single arrow (→), and when α changes from -∞ to 0 the root of the movement direction is marked with the double arrow.

Let's continue with the second equation

(5) $S^4+\alpha S-1=0$

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The root locus (RL) equation will be:

(6) $r^3 Sin3\varphi = -Sin\varphi,$

will be split into two equations:

$Sin\varphi = 0$

(7) $r^4(4Cos^2\varphi - 1) = -1.$

(The real roots of the equation and Complex roots of equations of motion trajectories).

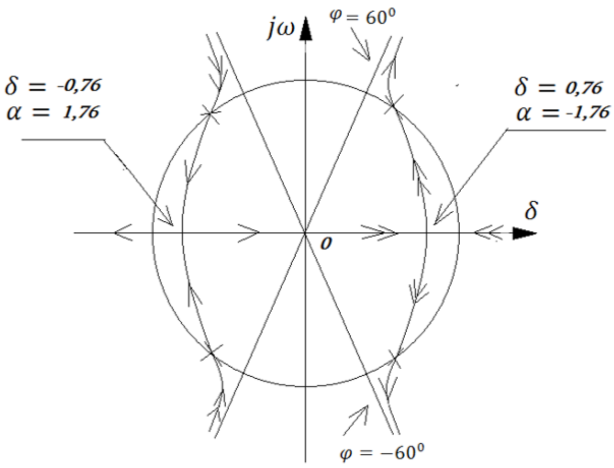

Figure 1. *Root locus for the equation* $S^4+\alpha S+1=0$.

The RL starting point will be $S^4-1=0$ equation roots: $S_1=e^{j0^o}$; $S_2=e^{j90^o}$; $S_3=e^{j180^o}$ and $S_4=e^{j270^o}$.

The RL areas will be:

$$\begin{cases}\sin 3\varphi > 0\\ \sin\varphi < 0\end{cases} \text{ an } \begin{cases}\sin 3\varphi < 0\\ \sin\varphi > 0\end{cases}$$

union inequalities system solutions (5)

$$\varphi \in]60^0; 120^0[\cup]-60^0; -120^0[.$$

(5) RL's chart depicted in “Fig 2.”

Now examine $S^4+\alpha S^2+1=0$ (8) equation roots trajectories. RL equation will be: $r^4 Sin2\varphi = Sin2\varphi$ so $Sin2\varphi=0$ or $r^2=1$. From here, $Sin\varphi=0$ (The real roots of the equation of motion) or $Cos\varphi=0$ From here, $\varphi=\pm\pi/2$ (the complex roots of the motion) and finally, $r=1$ or complex roots moving the unit circumference.

The equation (8) RL starting point will be: $S^4-1=0$ equation roots: $S^1=e^{j45^o}$; $S^2=e^{j135^o}$; $S^3=e^{j125^o}$ and $S^4=e^{j315^o}$. The real double roots $S=\pm1$, and the double complex root will be: $S=\pm j$; (8) RL's chart depicted in “Fig 3.”

Now establish $S^4+\alpha S^2-1=0$ (9) equation roots

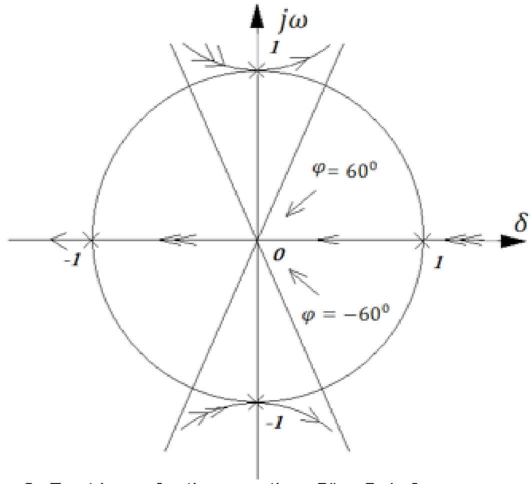

Figure 2. *Root locus for the equation* $S^4+\alpha S-1=0$.

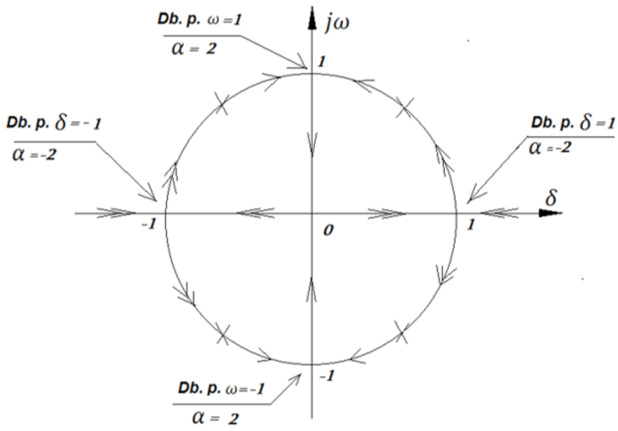

Figure 3. *Root locus for the equation* $S^4+\alpha S^2+1=0$

movement when α ├]-∞;+∞-┤[. RL equation would be: $r^4 Sin2\varphi = -Sin2\varphi$ (10), out of which $Sin2\varphi=0$ or $\varphi=90^O*k$; It turns out that equation (9) roots for any value of α is root's complex plane axis. The RL starting point will be: $S^4-1=0$ equation roots: $S^4=e^{j360^O k}$ so $S=e^{j90^O k}$. From here, we get 4 start points: $S_1=0$; $S_2=e^{j90^o}$; $S_3=e^{j180^o}$ and $S_4=e^{j270^o}$. (9) RL-equation are depicted in Fig 4.

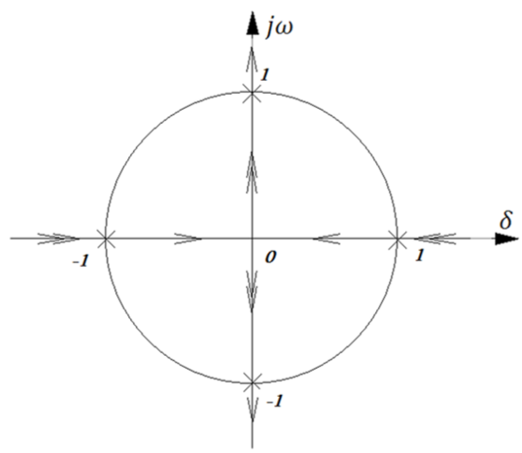

Figure 4. *Root locus for the equation* $S^4+\alpha S^2-1=0$

Conclusion

The study shows that the Root Locus method is the same for any order triplex equation.

We have proposed an expression of roots of a polynomial using trigonometric form to determine the properties and construction of the root locus. Such an approach is, at first glance, makes it difficult to achieve the desired result, but in fact the study of RL (algebraic equation root path) is defined so simply and beautifully, that this analytical approach to research and structuring graphs RL is undoubtedly reasonable and attractive.

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