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Catastrophe Predator-Prey Analysis with Root Locus

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Abstract

This article discusses the study of the Predator-Prey catastrophe with root locus. The method of research by root locus is much more evident in determining the extremity points for a different type of catastrophe.As a result of the research with root locus, we have found the extreme points of the catastrophe Predator-Prey function and their location on the complex plane, build the root locus and the catastrophecurve.

Keywords: Catastrophe Theory, Catastrophe Predator-Prey, Root locus (RL), Charts.

Introduction

The theory of Catastrophe is one of the major directions of modern science. It is widely used in various areas of science: mathematics, economics, physics, sociology, biology, etc. Catastrophe theory founder was Rene Tom and Christopher K. Zyman (20th century 60-70s).

There are generally different types of catastrophes: Catastrophe of the type Fold, Build-up catastrophe, Swallowtail catastrophe, Butterfly catastrophe, Predator-Prey catastrophe and others.

Various types of Catastrophes are described by differential equations. The equations parameters are changing; therefore, there is a bifurcation. The equation for one type of catastrophe Predator-Prey is:

(1) $$v(x,u,v) = \frac{1}{4}x^4 + \frac{1}{2}ux^2 + vx$$

In this equation x is variable and u and v are Coefficients that are changing. Our goal is to examine this model with root locus.

Methods

Root locus (RL) (gr. hodo - Road, Chart) is the unity of the algebraic equation roots' trajectory when the equation's coefficients, one or more, are changing.

Find the (1) equation derivative:

(2) $$v'(x,u,v) = x^3 + ux + v = 0$$

If we assume that root of the equation (2) is *x=r(cosφ+j sinφ)*, then equation of the root locus will be:

(3) r^3 *(cos3φ+jsin*^{3}*φ)+ur(cosφ+j sinφ)+v=0 =>*

r^3 cos3φ+jr^3 sin3φ+u r cosφ+j ursinφ+v=0

Real part of this equation is:

(4) *Re* r^3 *cos3φ+r cosφ+v=0*

The imagined:

(5) *Im* r^3 *sin3φ+ursinφ=0*

From the (5) determine the u:

u=-r^2*(sin3φ/sinφ)*

and insert in (4):

(6) r^3 *cos3φ-r3*(sin3φ/sinφ)*cosφ+v=0*

Multiply on sinφ:

r^3 *cos3φ sinφ-r3sin3φ cosφ+v sinφ=0*

From here we have:

-r^3 *(-cos3φ sinφ+sin3φ cosφ)=-v sinφ*

-cos3φ sinφ+sin3φ cosφ=sin(3φ-φ)=sin2φ

r^3 *sin2φ=v sinφ=>2*r^3 *sinφ cosφ=v sinφ*

*2*r^3 *sinφ cosφ-v sinφ=0=>*

*sinφ(2r3 cosφ-v)=0=>2*r^3 *cosφ= v and sinφ=0.*

cosφ=δ/r, where *δ* is *x*-axis and *r*– module, therefore *2*r^2 *δ=v*. Root locus located on the left half of the a flat surface, if *v>0* and the left, if *v<0.*This assertion comes from the fact that equation *2*r^2*=v/δ* is true, only in the case, when *δ> 0* and *v>0 or δ <0 and v<0.*

For the Root locus construction we use the equation [*n;m*] class system form. In particular

$P_n(S)+kQ_m(S)=0$.

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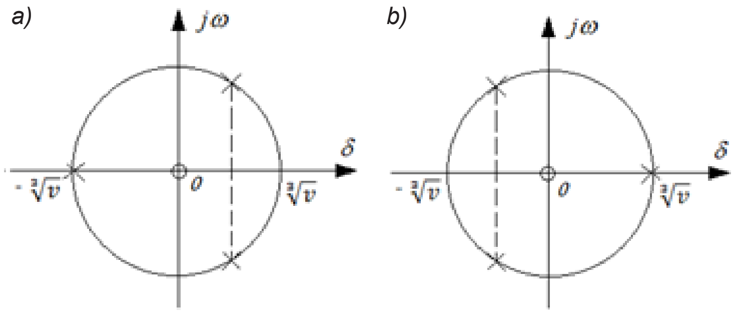

Figure 1. *Root locus starting points*

a) *δ>0 and v>0* b) *δ<0 and v<0.*

In our case, (2) will be recorded in the following equation:

$P_3(x)=x^3+v$ და $Q_1(x)=x(k=v)$.

Root locus starting points are the solutions of the equation:

(7) $x^3+v=0$

$x^3=-v$.If we consider the root record Euler formula $e^{j\omega}=\sin\varphi+j\cos\varphi$, then from (7) we have:

$$x_1=ve^{j(2n+1)} => x=3\sqrt{ve^{(j/3(2n+1))}}$$

Root locus starting points are:

$$x_1=\sqrt[3]{v}e^{j\frac{\pi}{3}}x_2=\sqrt[3]{v}e^{j\pi}x_3=\sqrt[3]{v}e^{j\frac{5\pi}{3}}.$$

Final point is: *x=0*. Chart. 1 a) and b) show the layout. The starting points are marked with "*x*" and the final points with "*o*."

Root locus double points can be found by the following formula:

$$\mathcal{P}_n(S)\mathcal{Q}_m(S)'+\mathcal{P}_n(S)'\mathcal{Q}_m(S)=0.$$

$$3x^2x-(x^3+v)=0$$

$$x=\sqrt[3]{\frac{v}{2}}$$

We determine the u at double point from equation (2).

$$u=-\frac{x^3+v}{x}=-\frac{\frac{v}{2}+v}{\sqrt[3]{\frac{v}{2}}}=-\frac{1{,}5v}{\sqrt[3]{\frac{v}{2}}};$$

From here:

$$u^3=-\frac{\left(\frac{3}{2}\right)^3+v^3}{\frac{v}{2}}=-\frac{\frac{27}{8}v^3}{\frac{v}{2}} => 4u^3=-27v^2$$

$$=> 4u^3+27v^2=0$$

RL start point is in ordinate centre, because *$Q_1(x)=x$ =>x=0.*

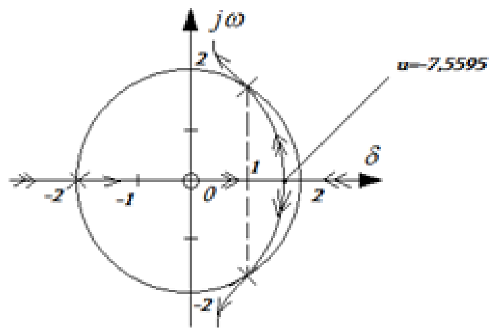

Figure 2. *Root locus for the equation $x^3+ux+v=0$*

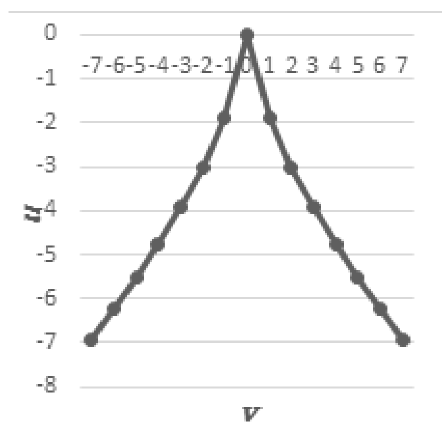

Figure 3. *Bifurcation curve $4u^3+27v^2=0$*

To build Root locus we use equality: $2r^2\delta=v$, where $r\geq\sqrt[3]{\frac{v}{2}}$. Then $\delta=\frac{v}{2r^2}$ and $\omega=\sqrt{r^2-\delta^2}=\sqrt{r^2-\frac{v^2}{4r^4}}==\frac{1}{2r^2}\sqrt{4r^6-v^2}$.

Root locus formula is: $\delta=\frac{v}{2r^2}$ and $\omega=\frac{1}{2r^2}\sqrt{4r^6-v^2}$.

Lets build RL when $v=8$.

From here changer with any step.

Root locus table is: $r\geq\sqrt[3]{\frac{v}{2}}=\sqrt[3]{4}=1{,}5874011,$

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Table 1. *Parameters of the root locus equation*

r	1,5874	1,7	1,9	2,1	2,3
δ	1,59	1,38	1,11	0,91	0,76
ω	0	0,99	1,54	1,89	2,17

Build the equation chart "Fig.3" shown the catastrophe area and bifurcationcurve. The Catastrophe area table is:

$$4u^3 + 27v^2 = 0 \Rightarrow 4u^3 = -27v^2 \Rightarrow u = -1{,}5\sqrt[3]{2v^2}$$

Table 2. *Catastrophe Predator-Prey parametrs values*

v	0	1	2	3	4	5	6	7
u	0,0	-1,9	-3,0	-3,9	-4,8	-5,5	-6,2	-6,9

Conclusion

The research shows that the root locus method simplifies finding the extreme points, catastrophe areas and building of the bifurcation curve in Catastrophe Theory.

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