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Solution of the Equation for the Axial Current Arising at the Surface of a Conducting Plate at a Plane Em Wave Scattering on it

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Abstract

The paper deals with the solution of two-dimensional integral equation for the surface axial current, arising at the surface of a conducting plate, when a plane EM wave scatters on it. The calculating formulae for the normalizing coefficient of the equation and for radiation characteristic of the plate are received.

Keywords: Plane EM Wave, Surface Axial Current, Radiation Characteristic.

Introduction

The problem of scattering of a plane EM wave at thin conducting plate is not yet strictly considered in the literature, while its cognitive and practical value is rather sufficient. The expression for the axial current arising at the surface of the plate in such conditions is considered and evaluated (Chikhladze, Kevanishvili, Kevanishvili &Kotetishvili, 2016). But now original mathematical approach to its solution should be found, in particular, definite coefficients of the equation have to be calculated. Their proper definition will become sufficient income in SHF and antenna techniques (Fig.1).

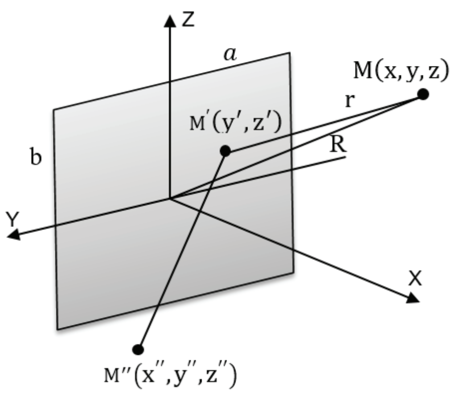

Figure 1. *Thin conductive plate, M radiation and M" observation points.*

The exact value for the surface axial current is given as follows (Chikhladze, Kevanishvili, Kevanishvili &Kotetishvili, 2016).

(1) $$J(y",0) = \frac{K(\alpha)}{\pi Z_0}\left\{\pi[\delta(\alpha + ky") + \delta(\alpha - ky")] + \frac{1}{\sqrt{\alpha^2 - k^2 y"^2}}\right\}.$$

Here $$K(\alpha) = \frac{2}{H_0^{(1)}(\alpha) - J_0\left(\frac{\alpha}{2}\right) H_0^{(1)}\left(\frac{\alpha}{2}\right)^2},$$

(2) $$\alpha = \frac{ka}{2}, \qquad k = \frac{2\pi}{\lambda}, \qquad Z_0 = 120\pi\,\Omega,$$

λ - wave length, δ - Dirac delta function, $H_0^{(1)}$ - Hankel function of the first kind, J_0 - Bessel function.

After definite transformations and insertions into (1) (Chikhladze, Kevanishvili, Kevanishvili &Kotetishvili, 2016), for the surface axial current arising at the surface of the plate we get

$$J(y",z") = -M + M\cos(kz) - Q\cos(kz'')\,J(y",0) + Q\sin(kz'')\cot(\beta)\,J(y",0) + {} + M\sin(k|z''|)\frac{1-\cos(\beta)}{\sin(\beta)}.$$
$$M = A(y'')\cos\beta + B(y'')\sin\beta,$$
$$\beta = kb/2 = \pi b/\lambda.$$

Obviously, this expression satisfies the $J\left(y', \pm {}^{b}\!/_{2}\right) = 0$ condition.

Solutions and Calculations

Let us count the value of "Q" coefficient, which is the normalized multiple in

(3) $$K(y' - y", z' - z") = Q\delta(y' - y", z' - z"),$$

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Introducing notations

$$y' = \frac{a}{2}\xi,\ \ y=\frac{a}{2}\xi,\ \ z' = \eta\frac{b}{2},\ \ z'' = \eta'\frac{b}{2}$$

Then overwrite (3) as follows:

(4) $$K\left(\frac{a}{2}|\xi-\xi'|,\frac{b}{2}|\eta-\eta'|\right)=Q\delta\left(\frac{a}{2}|\xi-\xi'|,\frac{b}{2}|\eta-\eta'|\right).$$

Introducing additional notations *$\tau_1=\xi-\xi'$, $\tau_2=\eta-\eta'$* conclude then that, as to *ξ,ξ^',η,η'* variables vary within the closed interval *($-1\le\xi,\xi',\eta,\eta'\le1$)*, *$\tau_1,\tau_2$* variables then should be placed inside *($-2\le\tau_1,\tau_2\le2$)* interval and (4) will be overwritten as follows:

(5) $$K\left(\frac{a}{2}\tau_1,\frac{b}{2}\tau_2\right)=Q\delta\left(\frac{a}{2}\tau_1,\frac{b}{2}\ \tau_2\right).$$

Integrating it by both *τ_1,τ_2* variables from -2 to 2 we get

(6) $$\iint\limits_{-2-2}^{22} K\left(\frac{a}{2}\tau_1,\frac{b}{2}\tau_2\right)d\tau_1 d\tau_2 = Q.$$

But (6) may be overwritten as

$$\iint\limits_{-2-2}^{22} K\left(\frac{a}{2}\tau_1,\frac{b}{2}\tau_2\right)d\tau_1 d\tau_2 = \iint\limits_{2-2}^{22} kC\frac{e^{-ik\sqrt{\left(\delta/2\right)^2+\frac{a^2}{4}\tau_1{}^2+\frac{b^2}{4}\tau_2{}^2}}}{\sqrt{\left(\delta/2\right)^2+\frac{a^2}{4}\tau_1{}^2+\frac{b^2}{4}\tau_2{}^2}}d\tau_1 d\tau_2.$$

Here *δ* is thickness of the plate.

Let us gain the circumstance that $K\left(\frac{a}{2}\tau_1,\frac{b}{2}\tau_2\right)$ function is finite from the physical point of view, in particular

$$K\left(\frac{a}{2}\tau_1,\frac{b}{2}\tau_2\right)\neq 0,\text{when } |\tau_1|,|\tau_2|\le 2,$$

$$K\left(\frac{a}{2}\tau_1,\frac{b}{2}\tau_2\right)= 0,\text{when } |\tau_1|> 2\ or\ |\tau_2|> 2.$$

That is why we may infinitely increase, say *τ_1* within the borders of integration, arriving then to the following (instead of (6)):

(7) $$Q = i\pi kC\int\limits_{-2}^{2} H_0^{(2)}\left(k\sqrt{\frac{\delta^2}{4}+\frac{a^2}{4}\tau_1{}^2}\right)d\tau_1$$

Here we used the relation

$$\int\limits_{-\infty}^{\infty}\frac{e^{-ik\sqrt{\frac{\delta^2}{4}+\frac{a^2}{4}\tau_1{}^2+\frac{b^2}{4}\tau_2{}^2}}}{\sqrt{\left(\delta/2\right)^2+\frac{a^2}{4}\tau_1{}^2+\frac{b^2}{4}\tau_2{}^2}}d\tau_2 = i\pi\int\limits_{-2}^{2}H_0^{(2)}\left(k\sqrt{\frac{\delta^2}{4}+\frac{a^2}{4}\tau_1{}^2}\right)d\tau_1.$$

The integral in the expression (7) may be calculated by any quadrature formula, for example, by the rectangular formula (Kevanishvili, Kotetishvili & Chikhladze et al., 2009) In given case, according to it, for three points *$\tau_1^{(1)}=-2,\tau_1^{(2)}=0,\tau_1^{(3)}=2$*, we arrive to the following result

(8) $$Q = i\pi kC\tfrac{4}{3}\left[2H_0^{(2)}\left(k\sqrt{\frac{\delta^2}{4}+4^6}\right)+H_0^{(2)}\left(k\,\delta/2\right)\right]$$

As to *$H_0^{(2)}$ ($k\,\delta/2$)* is a big number, in (2) all terms including *Q* are dominant and, thus, instead of (2) we write approximately

J(y",z")≈-QJ(y",0)×

(9) *×(sin(kz^") cot(β) - cos(kz")).*

This expression presents the approximate solution of the integral equation

$$\int\limits_{a/2}^{a/2}\int\limits_{b/2}^{b/2}J(y',z')K(y'-y,z'-z)dy'dz' = F(z",y")$$

being the more exact, the stronger is $k\,\delta/2 \ll 1$ (*) inequality.

Now let us count the radiation characteristic of the plate. First of all, we should write down the meaning of Green's formula e^{-ikr}/r in the far zone

$$\frac{e^{-ikr}}{r}\approx\frac{e^{-ikR}}{R}e^{-iky'sin\varphi sin\theta-ikz'cos\theta}$$

and insert this expression into the relation for Hertz's function given as follows:

$$\Pi_{1z} = C\int\limits_{-a/2}^{a/2}\int\limits_{-b/2}^{b/2}J(y',z')\frac{e^{-ikr}}{r}dy'dz'.$$

Resulting from it

$$\Pi_{1z} = C\frac{e^{-ikR}}{R}\times$$

(10) $$\times\iint\limits_{-a/2-b/2}^{a/2\,b/2}J(y',z')e^{+iky'sin\varphi sin\theta+ikz'cos\theta}dy'dz',$$

While foresee (9) we get

$$\Pi_{1z} = CQ\frac{e^{-ikR}}{R}\iint\limits_{-a/2-b/2}^{a/2\,b/2}(\sin(kz'')\cot(\beta)-\cos(kz''))\,(y",0)\times$$

(11) $\times e^{iky"sin\varphi sin\theta+ikz"cos\theta}dy$dz

J(y",0) being given by (1). Values of arising integrals are listed below *(β=π*b/λ)*

(12) $$f_1(\theta) = \int\limits_{-b/2}^{b/2}sinkz"e^{ikz"cos\theta}dz" = \frac{b}{2\beta}\left[\frac{\sin(\beta(1-cos\theta))}{1-cos\theta}-\frac{sin(\beta(1+cos\theta))}{1+cos\theta}\right],$$

(13) $$f_2(\theta) = \int\limits_{-b/2}coskz"e^{ikz"cos\theta}dz" = \frac{b}{2\beta}\left[\frac{sin(\beta(1-cos\theta))}{1-cos\theta}-\frac{sin(\beta(1+cos\theta))}{1+cos\theta}\right]$$

(14) $$f_3(\theta,\varphi) = \int\limits_{-a/2}^{a/2}J(y'',z'')e^{iky''sin\varphi sin\theta}dy" =$$

$$f_3(\theta,\varphi)\ = \frac{K(\propto)}{kZ_0}\left[2\cos(\propto\sin\varphi\sin\theta)+J_0(\propto\sin\varphi\sin\theta)\right],\propto=\ \frac{ka}{2}.$$

Now it is appropriate to write down (11) in the compact form

Π_{1z}=CQ e^{-ikR}/R [$f_1(\theta)$ctanβ-$f_2(\theta)$-$f_3(\theta,\varphi)$].

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Using the expression for electric field strength

$$E_{1z} = k^2\Pi_{1z} + \frac{\partial^2\Pi_{1z}}{\partial z^2},$$

it is possible now to calculate the electric field strength in the far zone. Taking into account that $\Pi_{1z}\sim 1/R$, while $\partial^2 \Pi_{1z}/(\partial z^2)\sim 1/R^2$, we cancel the second term in the right-hand side of this equation, then reducing it to the following:

$$E_{1z}=k^2\,\Pi_{1z}.$$

Conclusion

The practical interest is paid to the value of the azimuthal component of the electric strength $E_{1\theta}$, given as follows:

$$E_{1\theta} = E_{1z}sin\theta = k^2CQ\frac{e^{-ikR}}{R}F(\theta,\varphi), \quad F(\theta,\varphi) = [f_1(\theta)ctan\beta - f_2(\theta) - f_3(\theta,\varphi)]sin\theta \tag{15}$$

This is the radiation characteristic of the plate.

For numerical counting it is appropriate to normalize the expression (15) in $\theta=60^0$ and $\varphi=0$ directions. Then

$$F(60^0,0) = \left[\frac{b}{\beta}\left(sin\frac{\beta}{2} - \frac{1}{3}sin\frac{3}{2}\beta\right)ctan\beta - \frac{b}{\beta}\left(sin\frac{\beta}{2} + \frac{1}{3}sin\frac{3}{2}\beta\right) - T\right]\frac{\sqrt{3}}{2}, \quad T = \frac{3K(\propto)}{kZ_0}. \tag{16}$$

For the normalized characteristic we get:

$$F_{nor}(\theta,\varphi,\beta) = \frac{F(\theta,\varphi,\beta)}{F(60^0,0)}. \tag{17}$$

At the main meridian plane $(\varphi=0)$ the radiation characteristic will be represented as follows:

$$= \frac{K(\propto)}{\pi Z_0}\int_{-a/2}^{a/2} e^{iky''sin\varphi sin\theta}\left\{\pi[\delta(\propto + ky") + \delta(\propto - ky")] + \frac{1}{\sqrt{\propto^2 - k^2y"}}\right\}dy" \tag{18}$$

$$F_{nor}(\theta,0,\beta) = = \frac{[G(\theta)ctan\beta - f_2(\theta + g(\alpha))]sin\theta}{[T(\beta)ctan\beta - f_2(60^0) - f_3(60^0,0)]\frac{\sqrt{3}}{2}},$$

where

$$G(\theta) = \frac{\sin(\beta(1-cos\theta))}{1-cos\theta} - \frac{\sin(\beta(1+cos\theta))}{1+cos\theta}, \tag{19}$$

$$g(\alpha) = \frac{3K(\alpha)}{kZ_0} = f_3(\theta,\varphi), \tag{20}$$

Consider now the private case, when $\beta=(2n+1)\pi$ *or* $b=(2n+1)\lambda,(n=0,1,2,\dots)$, i.e. the height of the plate (b) [1] is the odd number of the wavelength. In such a case $ctan\beta=\infty$ and (18) sufficiently simplifies, transforming into the following

$$F_{nor}(\theta) = (-1)^n 2\frac{sin[(2n+1)\pi cos\theta]}{sin\theta}, \quad \frac{b}{\lambda} = 2n+1.$$

When $n = 0$, we get

$$F_{nor}(\theta) = 2\frac{sin(\pi cos\theta)}{sin\theta}, \quad \frac{b}{\lambda} = 1.$$

While, when $n = 1$

$$F_{nor}(\theta) = 2\frac{sin(3\pi cos\theta)}{sin\theta}, \quad \frac{b}{\lambda} = 3.$$

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