Analysis of Correction in Automatic Control Systems with Root Locus

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Abstract

The article addresses the major challenges of automated control systems and solutions. One way is to choose the appropriate correction for the system and analyze parameters. For the synthesis of corrective device parameters, the article uses the root locus method.Root locus can be constructed by changing one or more parameter, which allows a thorough analysis of the stability of the system.In article one of the types the integrating corrective scheme with r, L, C source of an input signal is considered. The problem of synthesis of parameters of an integrated corrective scheme with root locus is is solved; The root locus is constructed, and with their help, the possibility of the approach of roots of the characteristic equation, to roots of the ideal transfer function is considered.

Keywords: Control systems, automatic control system, correction, root locus, synthesis parameters.

Introduction

In the process of automatic control systems, engineers have to solve various problems. One of the most important things is the automatic regulation system (s) planning. These are: system power calculation, dynamics analysis, correction synthesis and experimental studies. After going through all these steps, The last phase is experimental research, that has its own difficulties. Often the calculations are correct, but the experiments can not show the result.

One of the reasons for this is that when selecting corrective parameters, they do not take into account. This leads to not the desired result. In general, information on corrective device is given in special tables. In automatic control systems should be possible to use the ready-correction schemes, but in reality, the case is different. The correction schemes in the tables is not considered a significant factor. In particular, any correction of the input signal source have resistance. This resistance is possible in some cases up to several hundred Oms. Also the input signal source internal resistance can be have both active and reactive resistance. Namely, the capacity and inductive.

Our goal is Take note the resistance of the input signal source during the calculations.

Methods

As it is known, the automatic control system planning process consists of three major steps:

1. Energy calculation;
2. Dynamic analysis and synthesis;
3. Experimental studies.

As a result of the energy calculation, the functional scheme is available. At this stage of planning is the selection of the basic elements. These include: control point adjustment setter or programmer software control systems, measuring transducer**,** preliminary and power amplifiers, realization mechanisms, regulators and functional sensors, calculation and execution of regulator and actuator and so on. As a result of the energy calculation, a functional scheme can be constructed by specifying the type or parameters of the elements. All elements of the scheme are selected according to technical

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requirements and are defined by properties of the regulated object.

The function chart as a result of energy calculation to remain invariable with its elements and is called an invariable part of a system of automatic control**.**

The initial data of the analysis of the dynamics of the synthesis are: the systems invariable part scheme of the energy calculation and the required technical specifications for the indicators of the quality of the dynamics.

As a result of the energy calculation of automatic control system in most cases is unstable or its dynamics is unacceptable.

For receiving the necessary dynamic process meeting the requirements of a technical task it is to make changes to the existing scheme. In the unchangeable part of the automatic control system, no adjustments can be made, so the only way to achieve the desired goal is to add an additional device, that will provide the required dynamic properties of the automatic control system.

In control theory such a device is the correction unit. In particular, the corrective device circuit must be constant; the use of this device should not cause technical and technological difficulties; turning correcting device should not reduce the reliability of the automatic control system.

In practice, corrective passive RC circuits are used. For these circuits, the transfer function parameters and calculation formulas logarithmic amplitude frequency characteristics (LAFC) are shown in special tables [1].

The final stage of planning an automatic control system is experimental research. One of the main tasks of this stage is to search for and adjust the parameters of corrective RC circuits, which provide the required dynamic process. This process has its difficulties. Namely, an experimental study of the parameters of the correction circuit may not give us the desired result and in most cases it is necessary to change the scheme. Sometimes with the help of experiments it is impossible to determine the correction schemes and parameters.

One of the important reasons for it is that data of tables of the corrective scheme (transfer function, LAFC, calculation formulas, etc.) do not contain resistance of an entrance source and of the output circuit which leads to errors.

The problem of the resistance is easily eliminated in practice. In particular, with the addition of resistance to the outcome circuit and its value should be as much as times the resistance of the input source. The problem lies in the resistance of the input source, the size of which often reaches several hundred ohms At the same time, induction and capacitive components of resistance do not take into account when calculating. If you do not take into consider these parameters, then there will be a significant deviation of the frequency and dynamic properties of the correction circuit, which are given in the tables of the correction circuits. At this time, the desired dynamic process cannot be achieved. There is a difference between real and desired dynamics and we are dealing with structural stability.

Question: What should be done to allow the synthesis of a corrective device resistance input source? Or, if compensation is possible?

To compensate for the resistance of the input source and solve the problem of dynamic synthesis, you can use the root locus (RL) method.

In this case, the transfer function of the correction scheme and the values of its parameters are unchanged.

Root locus method involves trajectory roots movement characteristic equation, when change one or more parameters. This problem can be solved by using the graph-analytic method of constructing RL.

As is known, for the problem synthesis a correction circuit linear automatic control system is the most commonly used frequency method (specifically logarithmic characteristics. As a result of a problem of synthesis logarithmic and amplitude-frequency (LAFC) characteristics of the correcting circuit according to which the scheme is selected from the table of correcting circuits.

Suppose that as a result of the synthesis of the corrective element by the log-amplitude-frequency characteristic. For the function of this circuit we have a transfer function as:

$$W(s) = \frac{(T_1S+1)(T_2S+1)}{(T_3S+1)(T_4S+1)} \qquad (1)$$

Where

$$T_1 = R_1C_1; T_2 = R_2C_2;\ T_3T_4 = T_1T_2;\ T_3+T_4 = T_1+\alpha T_2$$

and

$$\alpha = 1+\frac{R_1}{R_2}.$$

Corrective scheme and characteristics of parameters LAFC are given in “Fig. 1” and “Fig 2.”

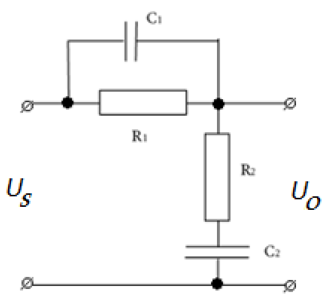

Figure 1

Correction circuit

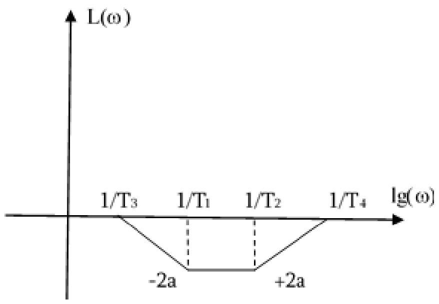

Figure 2

Logarithmic amplitude-frequency characteristic (LAFC) of the corrective circuit

(1) The transfer function given provided that the output resistance of the input U_S voltage source is zero, and output resistance is Infinite resistance. If the output operator for resistance input U_S voltage source is $r+LS$, and a resistance $R_o \to \infty$ the transfer function for “Fig.1” is:

$$W(s) = \frac{(T_1S+1)(T_2S+1)}{P(S)} \qquad (2)$$

Where

$$P(S) = T_1T_2\tau S^3 + (mT_1T_2 + T_2\tau + T_1T_2)S^2 + (mT_2 + nT_2 + T_1 + T_2)S + 1 \qquad (3)$$

$$\tau = \frac{L}{R_2};\ m = \frac{r}{R_2};\ n = \frac{R_1}{R_2}$$

If you compare functions (1) and (2), you can easily make sure that the polynomials are significantly different from each other, that is, there is a change in the structure of the correction circuit that is not taken into account when synthesizing this chain, that is, structural stability. In this situation, we are dealing with structural robustness [1,2].

The reliability of the automatic control system can be characterized by the ballast transfer function, often used in industrial automation:

$$W(S) = W_B(S)W_{ID}(S) \qquad (4)$$

where $W_B(S)$ is the transfer function of ballast:

$$W_{\mathbf{B}}(s) = \frac{(T_3S+1)(T_4S+1)}{P(S)} \quad (5)$$

Obviously, the ideal function of the correction circuit (1) should be $W_B(S)$ equal to one. Exact achieving this equality is impossible, but it can be approximated.

To do this, find a set of polynomial $P(S)$ coefficient values provide the roots of a polynomial: -1/T3, -1/T4 and S_0.

The coefficients of the polynomial must be chosen so that the root of S_0 could be the root of the imaginary axis on the left.

If these conditions are fulfilled then

$$W_{\mathbf{B}}(s) = \frac{1}{\tau S_0(S/S_0-1)} \quad (6)$$

Therefore $|S_0| >> \frac{1}{T_4}$, in formula (6) (S/S_0-1) can be ignored.

$$W_B(S) = \frac{1}{\tau S_0} \quad (7)$$

Thus, structural robustness was given parametric robustness, which is caused by (7) ballast. Only way change $P(S)$ polynomial roots in the desired direction and size, is to use a method of root locus.

From formula (3), it can be seen that the polynomial $P(S)$ can be changed by simultaneously changing the parameters τ, *m* and *n*.

After building these paths, we have the opportunity to find values, that give us the right distribution of the roots.

When analyzing the trajectories of the roots, as a result of changes in parameters τ, *m* and *n* are clearly visible distribution $P(S)$ polynomial roots. In addition, we can easily identify the parameters values, that will lead us to the desired result.

Sometimes the required values of the roots of $P(S)$ can be obtained in different values of the parameters. In this case, the parameters should be selected in terms of technical realization.

To construct the root locus, we can use analytical equations root locus, also as the graphical properties.

Let's consider two-element integrator corrective circuit. Scheme is shown in "Fig. 3."

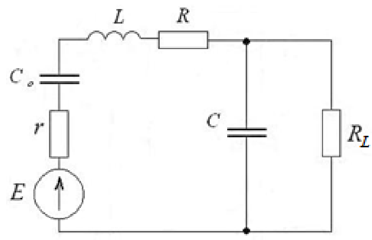

Figure 3

Integrator corrective circuit

The scheme uses the following indications: E - the input source signal EMF; r, L, C_0 - resistance components; R, C - the parameters of the chain; R_L - The stroke résistance of the ring. "Fig. 3" circuit transfer function is:

$$W_{EU_O}(S) = \frac{U_O(S)}{E(S)} = \frac{Z_2Z_L}{(Z_S+Z_1)(Z_2+Z_L)+Z_2Z_L} \quad (8)$$

$$W_{EU_O}(S) = \frac{R_{LL}C_0S}{\left[(r+R)C_0S+LC_0S^2+1\right]\left(R_LCS+1\right)+R_LC_0S} \quad (9)$$

$Z_S = r + LS + \frac{1}{C_0S}$; $Z_1 = R$; $Z_2 = \frac{1}{CS}$; $Z_L = R_L$. If $R_L \to \infty$ From (9) we get:

$$W_{EU_O}{}^*(S) = \frac{1}{\tau T' \cdot S^2 + T' \cdot S + \frac{1}{k}}, \quad (10)$$

where $T' = (r+R)C; \tau = \frac{L}{r+R}; k = \frac{C_0}{C_0+C}$

Examine (10) transfer function poles trajectories, when τ is a change of the $[0;+\infty)$ interval.

If $\tau = 0$, then $S = -\frac{1}{kT'}$; If $\tau \to \infty$, then $S = 0$;

Root locus Equation is:

$$\left[\mathcal{P}_n(\delta) - \frac{\omega^2}{2!}\mathcal{P}_n''(\delta) + ...\right]\cdot\left[Q_m(\delta) - \frac{\omega^2}{3!}Q_m'''(\delta) + ...\right] - \\ -\left[\mathcal{P}_n'(\delta) - \frac{\omega^2}{3!}\mathcal{P}_n'''(\delta) + ...\right]\cdot\left[Q_m(\delta) - \frac{\omega^2}{2!}Q_m''(\delta) + ...\right] = 0 \quad (11)$$

If we use for the formula (10) the [n;m] class equation system:

$$P_n(S) + KQ_m(S) = 0 \quad (12)$$

in this case $\mathcal{P}_2(S) = T' \cdot S^2$ and $Q_1(S) = T' \cdot S + \frac{1}{k}$

Given this RL equation we get the following equation:

$$\left(\delta + \frac{1}{kT'}\right)^2 + \omega^2 = \left(\frac{1}{kT'}\right)^2 \quad (12)$$

The graph of this equation is a circle centered $\left(-\frac{1}{kT'};0\right)$ and radius $\rho = \frac{1}{kT'}$ "Fig. 4."

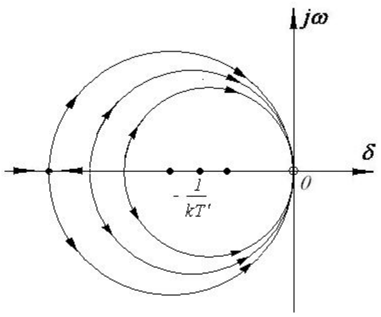

Figure 4

Root locus Integrator corrective circuit

The real double point of the root locus is $S = -\frac{2}{kT'}$

The parameter τ value in this point is: $\tau = 0{,}25kT'$.

[1,2]. Points with the greatest imaginary part of the root locus are: $S = -\frac{1}{kT'} \pm j\frac{1}{kT'}$. In these points $\tau = 0{,}5kT'$

The ideal transfer function for the integral link is:

$$W_{EU_O}{}^{ID}(S) = \frac{1}{TS+1} \quad (13)$$

Therefore, (3) the pole of the transfer function should be: $S = -\frac{1}{T}$ and the second pole must be in the imaginary axis, from left to far more than the pole $S = -\frac{1}{T}$.

The condition will be fulfilled, If the starting point $S = -\frac{1}{kT'}$ is close to the desired pole $S = -\frac{1}{T}$.

Suppose that the distance between the starting pole and to the desired pole ($S = -\frac{1}{T}$ and $S = c$) is equal distance $\mu\%$. This condition is written in the form of the equation:

$$-\frac{1}{kT'} + \frac{1}{T} = \frac{\mu}{100}\frac{1}{T}. \quad (14)$$

From here

$$kT' = \frac{100 \cdot T}{100 - \mu} \quad (15)$$

Conclusion

The analysis proves the structural robustness of two-element integrating chain; structural robustness is compensated by correction chain transfer function's root locus.

The method of a root locus can be used in control systems at the change of parameters of the characteristic equation. However, it is admissible only if we use root locus diagrams together with their analytical at.

This method can be applied to different types of corrective schemes: differential, integrator and integrator-differential which have different types of resistance of sources of an input signal.

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