About the Methodology of Teaching the Moment of Inertia in Algebra Based Physics Course

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Abstract

In University physics courses the moment of inertia is a new concept, which falls out from secondary school level. The article presents the discussion of how the "moments of inertia" can be treated without using calculus in algebra based introductory physics courses. The proposed methodology defined in algorithmic way is introduced: representing the initial object as sum of discrete N masses, that is transferring the problem from continuous to discrete one; providing summation over such a discrete system to calculate the moment of inertia of it I_N ;tending N to infinity , that is returning from discrete to continuous mode again to obtain $I = lim_{N\to\infty} I_N$. The proposed methodology relies on the summation techniques readily accessible to an algebra-based introductory physics class. In fact, what students are required to know from mathematics, are some of the formulas for sum of degrees of the natural numbers: $\sum_1^N k^m \quad (m = 1,2,3, \dots)$ Deriving that is within the scope of students' abilities: as prerequisite, all what they have to know is formula for the sum of arithmetical progression for the first N natural numbers, and the formulas for higher degrees can be obtained from it by simple recurrence technique. Based on few examples, it is shown how the proposed algorithmic methodology works in practice for situations of various shaped bodies.

Keywords: Methodology of teaching, Moment of Inertia, Non-calculus based physics course, Algorithm based on summing of degrees of natural numbers.

Introduction

The algebra based physics courses are targeted usually for science or science related undergraduates that do not require physics beyond the introductory level. At present at IBSU physics course is delivered for Computer Technology and Engineering faculty freshmen students as the algebra based introductory one/first semester course.

Avoiding calculus limits the deeper understanding of various physical aspects. Students remain at the secondary school physics level. Usually, methodology of "calculus"-type thinking is based on development of graphical representation of various non-constant quantities and describing the desired quantity as the graph area. Such approach, though very useful for gaining insight into calculus-specific peculiarities, couldn't be extended to any situation. The "moment of inertia" is one of the most distinguished from this point of view.

In University physics courses the "moment of inertia" is a new concept, which falls out from secondary school level. The source of problem is that the rotational inertia of an object depends not only on its mass, but also as how that mass is distributed with respect to the rotational axis. Usually this discussion is avoided and students are given standard formulas. The vague statement is made that it is a matter of calculus to obtain them.

The goal of the present paper is to discuss the methodology of how the "moments of inertia" can be treated without us ing calculus. In fact, what students are required to know from mathematics, are some of the formulas for sum of degrees of the natural numbers: $\sum_1^N k^m \quad (m = 1,2,3, \dots)$ (Korn and Korn, 1968). Deriving them is within the scope of students' abilities: as prerequisite, all what they have to know is formula for the sum of arithmetical progression for the first N natural numbers, and the formulas for higher degrees can be obtained from it by simple recurrence technique. For example, for m=2, m=3 and m=4 they have the form

$$\sum_1^N k^2 = \frac{N \cdot (N+1) \cdot (2N+1)}{6} \qquad (1)$$

$$\sum_1^N k^3 = \frac{N^2 \cdot (N+1)^2}{4} \qquad (2)$$

$$\sum_1^N k^4 = \frac{N \cdot (2N+1) \cdot (3N^2 + 3N - 1)}{30} \qquad (3)$$

So students feel themselves quite comfortable with it.

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The Methodology

The most usual example, discussed in introductory algebra based courses, is the moment of inertia of a thin ring with mass m and radius R, rotating about an axis through its center.

Representing the ring as sum of point-like masses each located at the same distance from the axis of rotation, the "moment of inertia" is given in the form

$$I = \sum_{i=1}^{N} m_i R^2$$

Further the R^2 term is factored out, giving in result

$$I = R^2 \sum_{i=1}^{N} m_i = mR^2$$

In fact, the underlying idea of methodology, proposed below, is already sparked in such approach.

So, the methodology, clarified and stated more precisely in algorithmic way, is following:

- Represent the initial object as sum of discrete N masses that is transfer the problem from continuous to discrete one.
- Provide summation over such a discrete system , that is calculate the moment of inertia of it I_N
- Tend N to infinity$N \to \infty$ that is return from discrete to continuous mode again to obtain $I = \lim_{N\to\infty} I_N$

What we are going further, is to show how the proposed algorithm works for situations of various shaped bodies.

Examples

Example 1: Finding the moment of inertia of the long uniform rod of length L and mass m, rotating through its center.

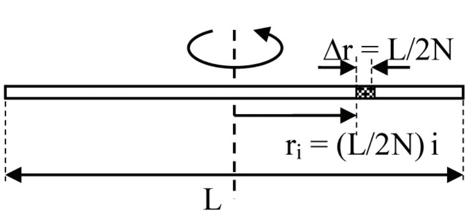

Figure 1

As the first step, due to algorithm, make discrete version of the object. We can represent the rod as a sum of point-like masses, each equal to $^m/_{2N}$, evenly spaced, at locations given by

$$r_i = \frac{L}{2N} i.$$

We also can divide the full length into two, due to symmetry, and sum other one part, multiplying the result by factor 2 (See the Fig.1).

So, due to definition, moment of inertia of such object is represented as

$$I_N = 2 \cdot \frac{m}{2N} \cdot \left(\frac{L}{2N}\right)^2 \sum_{i=1}^{N} i^2$$

Using formula (1), this is transformed to the form

$$I_N = \frac{mL^2}{4N^3} \frac{N(N+1)(2N+1)}{6}$$

Taking the limit $N\to\infty$ that is going from point-like to uniform distribution, the moment of inertia of uniform rod is obtained

$$I = \frac{mL^2}{12}$$

Surely, the result is the same as obtained via integration (Yavorsky and Detlaf , 1977).

Example 2: Finding the "moment of inertia" of a rectangular thin plane of uniform composition, of length L and width W, with location of axis through its center.

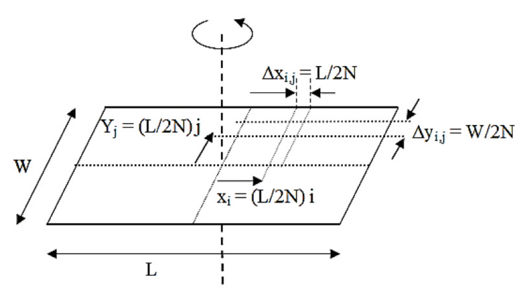

Figure 2

Also, as the first step, compose the discrete version of the object. Divide the area of chosen quadrant into the partial rectangular ones $\Delta x_{i,j}$ and $\Delta y_{i,j}$. We use freedom to make choice of equidistant partition across the axis X and Y,

say 2N×2N, so that

$$x_i = \frac{L}{2N} i, \; y_j = \frac{W}{2N} j \quad \text{and}$$

hence,

$$\Delta x_{i,j} = \frac{L}{2N} , \; \Delta y_{i,j} = \frac{W}{2N} ,$$

where the i index spans over X axis, and j - over Y axis. (See the Fig. 2).

For this object we can also divide the full area into four, due to symmetry, and sum other one quadrant, multiplying the result by factor 4.

The partial masses can be evaluated as $m_{i,j} = h \cdot \rho \cdot \Delta x_{i,j} \cdot$

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$\Delta y_{i,j}$,

so we obtain that

$$m_{i,j} = h \cdot \rho \cdot \frac{L \cdot W}{4N^2} = \frac{m}{4N^2}.$$

(We use the constraint for the mass of the ring m=h·ρ·L·W).

Due to definition, moment of inertia is

$$I = \sum_{i,j} m_{i,j}\ (x_i^2 + y_j^2)$$

So, after doing some simple algebra, one yields

$$I = 4 \cdot \frac{m}{4N^2} \sum_{i,j} (x_i^2 + y_j^2) =$$

$$\frac{m}{N^2}\left(\frac{L^2}{4N^2}\sum_{i,j}^{N} i^2 + \frac{W^2}{4N^2}\sum_{j,i}^{N} j^2\right) =$$

$$\frac{m \cdot (L^2 + W^2)}{4N^4} \cdot \left(N \cdot \sum_{i=1}^{N} i^2 + N \cdot \sum_{j=1}^{N} j^2\right)$$

Here the double sums have been reduced to multiple of factor N and single sum of square of natural numbers from 1 to N. Using formula (1) and rearranging the N dependence, one obtains

$$I = \frac{m \cdot (L^2 + W^2)}{12} . (1 + \frac{3}{N} + \frac{1}{N^2})$$

Taking the limit N→∞, we get anticipated result

$$I = \frac{m \cdot (L^2 + W^2)}{12}$$

The same, as obtained usually via integration (Yavorsky and Detlaf , 1977).

Example 3: Finding the expression for the "moment of inertia" of solid thin cylinder of uniform composition, of radius R and height h with location of axis through its center.

As the first step, discrete representation is arranged by fragmentation of the radius of the ring by N and representing the body as equidistant thin circular rings. That is, we make use of radial symmetry and introduce partial radius $r_i = \frac{R}{N} \cdot i$.(See Fig. 3)

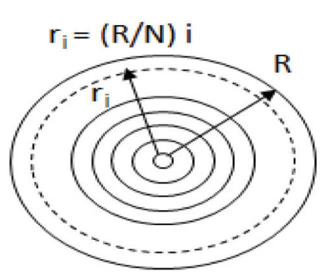

Figure 3

Due to definition, moment of inertia of such discrete object is the sum

$$I_N = \sum_i \Delta m_i \cdot r_i^2$$

The mass of i-the partial ring is $\Delta m_i = \Delta S_i \cdot \rho \cdot h$.

Since $S_i = \pi \cdot r_i^2$, the difference

$$\Delta S_i = S_i - S_{i-1} = \frac{\pi R^2}{N^2}(2i - 1)$$

Substituting expressions of partial radius and mass into initial formula gives

$$I_N = \frac{m}{N^2} \cdot \frac{R^2}{N^2} \sum_{i=1}^{N} (2i-1) \cdot i^2 = \frac{mR^2}{N^4}\left[2 \cdot \sum_{i=1}^{N} i^3 - \sum_{i=1}^{N} i^2\right]$$

Using summation formulas (1) – (2) leads to

$$I_N = mR^2(\frac{1}{2} + \frac{1}{2N} - \frac{1}{2N^2} - \frac{1}{3N^3})$$

Taking the limitN→∞, we get

$$I = \frac{mR^2}{2}$$

The anticipated result, same as obtained usually via integration (Yavorsky and Detlaf , 1977).

Conclusion

Based on few examples, we have shown how the proposed methodology works in practice. But it is not limited to discuss examples and can be easily used for other types of shaped bodies, usually presented at physics courses. We want to stress that proposed methodology relies on the techniques readily accessible to an algebra-based introductory physics class. As a positive outcome, it should be mentioned, that using the proposed methodology at classes develops in students calculus type thinking. So it is beneficial not only for treating of the problem moment of inertia of the body, but for gaining insight and understanding of various topics of physics subject there the distribution of physical quantity is crucial.

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