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A Mechanical Analogy to Optical Phenomenon for General Physics Courses

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Abstract

The brief review is given about the role of analogues in teaching practice. Further, the new mechanical - optical analogy is proposed for a phenomenon of the propagating short, intense, and coherent laser pulse through optically resonant medium of atoms (Self Induced Transparency effect) and on the passage of massive ball through the system of pendulums. The analytical solutions are derived for mechanical model, which is within the scope of student's abilities: as prerequisite, all what they have to remember is treating collisions in one-dimensional situation based on conservation laws of energy, momentum, and linear momentum. The derived solutions explicitly indicate the similarity between passing (under some constraints between the parameters of system) of laser pulse through the medium and massive ball through the system of pendulums without loss of energy. The positive outcomes for teaching process are briefly discussed.

Keywords: Mechanical analogy to optical phenomenon, momentum, Self Induced Transparency effect, physics courses, laser pulse.

Introduction

Analogy in a nutshell → Everyday observations reflect the fact of the primary way humans learn, think and communicate in general is by analogies. The pedagogic value of analogical reasoning is that it allows us explain the unfamiliar in terms of the familiar ones. But many scientists involved in artificial intelligence studies, go further and claim that analogies are more than convenient terminology, but **"Analogy-making is at the core of human cognition"** (Hofstader, 2001; Speed).

Due to general definition, an analogy is a mapping of relations from objects in a base domain to objects in a target domain. One of the well-known works on analogy gives the following standard definition (Gentner, 1983): "The analogy 'a T is like a B' defines a mapping from B to T". B is called the base domain and serves as a knowledge source. T is called the target domain, and is the subject to be learned. Symbolically, the analogy is the mapping M, M:bi → ti

The use of analogies is an important element in teaching science in general, and physics in particular. Analogy making is a powerful technique used itself by all practicing teachers. Science classes are full of abstract or challenging concepts that are easier to understand if an analogy is used to illustrate the points. Effective analogies motivate students, clarify students' thinking, help students overcome misconceptions, and give students ways to visualize abstract concepts. Teachers use analogies to build conceptual bridges for students between what is familiar (an analog concept) and what is new (a target concept). The Teaching with Analogies Model (Glynn,1995) includes following sequence of steps: Introduction of the target concept; Review the analog concept; Identify relevant features of the target and analog; Map similarities; Indicate where the analogy breaks down; Draw conclusions.

Following is list of some analogies commonly presented in introductory physics courses (Giancoli, 2005; Halliday, Resnick & Walker, (1993)) The "soldier" analogy of refraction at an interface between media of different refractive index, "A mechanical model with transverse waves passing through vertical and horizontal slits to conceptualize the polarization of light", "The hydraulic analogy of electric currents", "The analogy between water waves (in a ripple tank) and light waves", "Coulomb's law analogy with Newton's law of gravitation", "The electric field's analogy with a temperature field", "Analogy of storing energy in a capacitor and in stretched spring (or lifting a book)", "Analogy between emf device and a charge pump"; "Analogy of the earth with a huge magnet"; "Analogy between the electric circuit consisting of an inductor, capacitor and resistor and a viscous system of a mass on spring"; "Analogy between the solar system and the atom, with the Sun being analogous to the nucleus and the planetary orbits to electronic ones"; "Analogy that particles are like sending a letter, while waves are like making a telephone call".

From the above one can deduce that analogies occurring in texts may, first, be simple-based on surface similarities, like some type of metaphor, and second, deeper and rigorous - based on similarities expressed in similar types of mathematical equations.

It is obvious that mechanical analogies, due to their role in human's everyday life, are valuable. Maybe the

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most striking mechanical analogy of second type in physics is correspondence between a mass on a spring (simple harmonic motion) and a wide variety of physical systems spanning from classical physics to quantum physics and quantum field theory (Moshinsky and Smirnov, 1996). Simple harmonic motion it's how we understand the motion of a pendulum, or the vibration of an extended object; it's the basis for the simplest model of the propagation of sound and heat through a solid; it's the starting point for models of vibrating molecules; theory of light as a quantum field comes from making the electromagnetic field look, mathematically, like a mass on a spring; string theory itself comes from the correspondence between the behavior of mathematical objects describing particles and the mathematical description of a vibrating string, which is, itself, a variant of a mass on a spring (Orzel).

Despite the fact, that using analogies became an indispensable element of modern teaching methodology, one can find a large volume of scientific literature, devoted to the dangers of thinking by analogy (Simanek,). Without going into details, here we claim, that we agree with the opinion that such literature is a reflection of misunderstanding of the main essence of analogues (Muldoon, 2007). Indeed, the bad analogies can cause problems, but that's due to the badness of the analogies, not an inherent weakness of the method of explaining complex things by comparing them to simpler and more familiar things. One should always remember that only analogues, based on similarities expressed in similar types of mathematical equations, can be pursued more deeper and can lead to rigorous scientific results, while the simple-based on surface similarities, like some type of metaphor, has limited range of their application and it is inappropriate to pursuing them in details out of the scope of their validity.

In this paper we are going to construct a mechanical analogy to optical phenomenon of the propagating short, intense and coherent laser pulse through optically resonant medium of atoms (Self Induced Transparency effect), based on passage of massive ball through the system of pendulums.

Base domain. We will choose the phenomenon from the topic of propagation of electromagnetic field through the medium. We will concentrate on the passage of the very bright, very short light pulses through an optical material, mainly on the so-called phenomenon of Self Induced Transparency (SIT) effect (McCall and Hahn (1967), (1969)), which along with its fundamental interest has also many interesting and useful technological applications. The effect is due to interplay of various constrains on frequency, amplitude, and duration of pulse, resulting in the fact that propagation of coherent laser pulse in material which normally absorbs light, can become completely transparent to a bright, short-duration light pulse.

Let's discuss the SIT mechanism without going in-depth peculiar details (Allen and Eberly, 2005). It is a phenomenon in which a pulse of coherent light, with a certain frequency, amplitude, and duration, is transmitted by a normally opaque medium. Short duration means that duration of pulse is much less than relaxation times $\tau p << T1, T2$ (τp -duration of pulse, T1 – lifetime of exited state of atom of media, T2 – polarization relaxation time, which characterize decay rate of the dipole moment of the system; Usually $T2 << T1$). If the field intensity is high enough, the ensemble of resonance atoms under the action of the first half of pulse (at the pulse edge); are converted (due to induced absorption) to the coherent excited state and, further, under the action of the second-half of pulse (on the decrease in the pulse) they coherently relax (due to induced emission) into the ground state. Thus, the radiation is not absorbed.

SIT is optical representation of concept of soliton, which in various branches of modern mathematics and physics means a self-reinforcing solitary wave (a wave packet or pulse) that maintains its shape while it travels at constant speed (Lamb, Jr., 1971).

Mathematical description of the phenomenon of self-induced transparency is based on the self-consistent solution of the equations of Maxwell-Bloch: Maxwell wave equation is responsible for the distribution of light pulse (classical equations) in the media of resonant two-quantum-level optical oscillators, the dynamics of which is determined by the optical Bloch equations (quantum equations).

The effect of SIT theoretically was predicted, verified, and proved experimentally. At present, it is well-established effect with impact on various technological branches.

It is a complex effect, based on a number of physical concepts, inclusion in material and discussion of which will be of great pedagogical profit when teaching students electromagnetism. It bridges macroscopic and microscopic views of propagation of light through medium and incorporates many important physical concepts and quantities, both macroscopic and microscopic, such as Maxwell's equation for light propagation, lasers, coherence, short pulses, spontaneous decay coefficient, induced absorption and emission, resonant absorption, representation of atom as two level systems, two level quantum oscillator. However, at present one cannot find this effect mentioned in textbooks of general physics, even in more advanced ones. To our opinion, this fact is due to the mathematical complexity of the SIT effect. So, what we are going to do is to introduce a mechanical analogy to SIT effect, which will simplify understanding of this phenomenon by students.

Target domain. As a model we will discuss here the passage of massive ball through the system of pendulums. See the "figure1" below:

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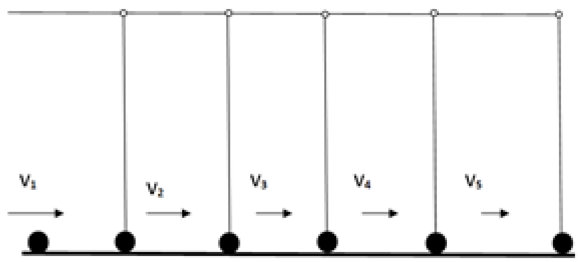

Figure 1: *Passage of massive ball through the system of pendulums.*

I level

For general understanding, some constraints are adopted: Motion is one-dimensional; ball is sliding, with no rotation. System is isolated, where is no friction between ball and surface and at fixing points of pendulums; pendulums are hard rods, with negligible mass (moments of inertia I = 0); the whole mass of pendulum is concentrated in the attached spring; collision is fully elastic. The diagram for first collision is depicted below:

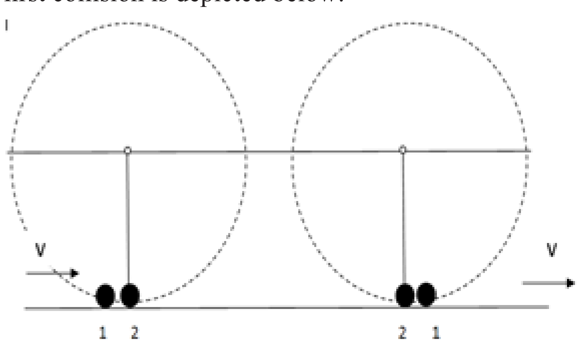

Figure 2: *An elementary act of the first ball – pendulum collision*

Students are familiar with one-dimensional collision from the course of mechanics[1], so they easily accept the following equations, based on momentum and energy conservation of system:

$$mv = MV + mu \tag{1}$$

$$\frac{mv^2}{2} = \frac{MV^2}{2} + \frac{mu^2}{2} \tag{2}$$

Here v is initial velocity of colliding ball, m – it's mass, $\boldsymbol{u}$ - it's velocity after collision, V – velocity of pendulum ball after collision and M - it's mass.

Solving them in V and u one obtains:

$$V = \frac{2v}{\mu+1} \qquad u = v\frac{1-\mu}{\mu+1} \tag{3}$$

Here (μ=M/m)

Let us discus the results:

• For $\mu < 1$ the ball after collision acquires negative velocity and fails to pass;

• For $\mu > 1$the ball continues motion with reduced velocity, while the first pendulum starts oscillation. Therefore, the energy of the ball is reduced (analogy to dissipation of electromagnetic radiation in medium); after the first collision, the process will continue eventually, transferring initial energy to pendulum oscillations. So the initial energy is redistributed. In final, the ball will stop and stuck in pendulum system or exit with reduced energy.

• For μ =1 we yield an unique solution: the pendulum ball starts rotation with $\boldsymbol{V= v}$ velocity, while the colliding ball stops with $\boldsymbol{u=0}$. To discuss the process in details, we have to write down once more the collision equations, with change, that now the pendulum ball strikes the initial ball, which is at rest after first collision. Due to the neglect of inertia moment, the problem is similar to initial one and it can be easily solved, or deduced, that pendulum ball stops, while the colliding ball continues motion with initial v velocity. After this, the process will become periodic: in every elementary act of collision, the energy transferred to pendulum system is fully transferred back to colliding ball, which will continue the motion with the initial energy, and, hence, without energy dissipation.

II level

For more advanced students. Now we discuss more realistic model of pendulums – we treat them as massive rods with uniformly distributed mass of M. See the figure 3 below.

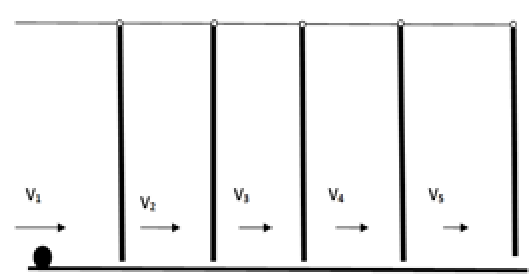

Figure 3: *Representing pendulums as massive rods.*

1 Any standard course will serve, for example Giancoli, 2005; Halliday, Resnick & Walker, (1993) Equations, to handle the collision process are based on conservation laws of linear momentum and energy of the system:

$$mvl = I\omega + mul \tag{4}$$

$$\frac{mv^2}{2} = \frac{I\omega^2}{2} + \frac{mu^2}{2} \tag{5}$$

$$I = \frac{Ml^2}{3} \text{ and } \omega = V/l$$

Here "***I***" is the momentum of inertia of pendulum, and "***ω***" is angular velocity of the edge point of pendulum. As in above set of equations, (1) and (2), these set of equations are written for the instant of collision process.

Solving them in V and ***u*** one obtains:

$$V = \frac{6v}{\mu+1}\ , u = v\frac{1-\mu}{\mu+1} \tag{6}$$

After the first collision, the process will continue eventually, and solutions can be determined by reiteration of collision process equations. For us it is remarkable, that the main results of the I level model is sustained:

- For $\mu = 1$ we yield a unique solution: the pendulum starts rotation with ***V=3v*** velocity of edge point, while the colliding ball stops ***u=0***. Afterwards the pendulum strikes the initial ball, been stopped and the colliding ball will continue motion with initial v velocity. As discussed above, after this the process will become periodic: in every elementary act of collision the energy transferred to pendulum system is fully transferred back to colliding ball, which will continue the motion with the initial energy and, hence, without energy dissipation.

Discussion

We have constructed a mechanical analogy to optical phenomenon of the Self Induced Transparency, based on simple mechanical model of massive ball passing through the medium of mechanical pendulums. It was shown, that the essence of both effects, mechanical and electromagnetic, is due to peculiar constraints on the parameters of the process – the field intensity, frequency, amplitude, pulse duration for electromagnetic one, and initial velocity, mass ratio and distance between pendulums for the mechanical one.

Therefore, what do we expect from the proposed analogy? Mapping the complex mechanical analogy with simpler mechanical one is not an end in itself. We think that it has also some positive outcomes for teaching process too. We think the proposed analogy can bridge the gap for students between quantum and classical description of matter and electromagnetic radiation, motivate and promote their interest in modern technologies based on lasers, enhance their skills of modeling physical situations and stimulates their inventory reasoning.

Conclusion

The brief review is given about the role of analogues in teaching practice. Further, the new mechanical - optical analogy is proposed for a phenomenon of the propagating short, intense, and coherent laser pulse through optically resonant medium of atoms (Self Induced Transparency effect) and on the passage of massive ball through the system of pendulums. The analytical solutions are derived for mechanical model, which is within the scope of students' abilities: as prerequisite, all what they have to remember is treating collisions in one-dimensional situation based on conservation laws of energy, momentum, and linear momentum. The derived solutions explicitly indicate the similarity between passing (under some constraints between the parameters of system) of laser pulse through the medium and massive ball through the system of pendulums without loss of energy. The positive outcomes for teaching process are briefly discussed.

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