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Iterated SVD for Improving Spectral Resolution of Nonstationary Signal

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Abstract

It is well known and proved already that principal singular vectors of Hankel data matrix repeats same spectral structure and by concatenation, these singular vectors allows us to gain high statistical stability and at the same time improve spectral resolution, but the main question arise; are they working well when frequencies are separated well enough or can they detect hidden periodicities even behind the resolution limit? In this article iterated SVD method is used to increase length of the time series much bigger than single SVD can do and at the same time increase spectral resolution and decrease noise in a new time series at more level then only one SVD can do.

Keywords: Iterated singular value decomposition, spectral resolution, SNR (signal to noise ratio), PSD (power spectral density)

Introduction

Spectral resolution is one of the biggest problems in digital signal processing and signal analyzing, while in Business sciences, forecasting and modeling of given time series is very crucial among large variety of random processes (having trend, stationary, ergodic, purely random, etc.), processes containing periodical and random components are also very important (A.Milnikov). Correct estimation of parameters, as well as ability for resolving capacity or ability to separate components, which are close to each other in amplitudes and frequencies, are very important to model data correctly and finally to get accurate result for future analysis and forecasting.

Literature Review

There exist several approaches for spectral estimation in modern literature, including parametric (Auto Regressive (AR), Autoregressive Moving Average (ARMA) and non-parametric models, while these methods work well in many cases, they all have basic limitations. Parametric method requires some prior knowledge about time series (for instance it can be presented by autoregressive model or it can be stationary series by finite differencing) (Castanié, 2006), another problem of parametric method is selection of correct order of AR/ARMA model, at the same time these methods are sensitive to noise and in case of presence of noise, it can give us incorrect result. Another variant of AR model is a Subspace method, which requires number of parameters to be known in advance (Hayes, 1996). On the other hand non-parametric method lacks the possible information about signal, and also it is not statistically stable, for example, Fourier method does not work well for non-stationary and non-harmonic series, while periodogram has variance problem (Hayes, 1996), there exist several approaches to reduce variance in periodogram method, but it will also decrease it's spectral resolution ability.

Problem Statement

In this article we are considering the following model for analyzing given signal represented by some number of periodic components with additive white noise

$$x(t) = \sum_{j=1}^{m} A_j e^{(2\pi f_j t + \theta_k)i} + w(t) \qquad (1)$$

In this model amplitudes, frequencies and phases are fixed and according to (Marple, 1987), this model represents non- stationary signal. The ratio of frequencies can be any number; it is not required that frequencies be harmonically related to each other. The main problem of spectral estimation method is to effectively separate components from each

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other and to get good spectral pictures. Various parametric and non-parametric methods fail in this task. Recently it was proved that spectral resolution and resolving capacity can be improved by concatenating principal singular vectors of Hankel data matrix after singular value decomposition and therefore increase total observation time, which will help us to identify components that are very close to each other (A.Milnikov), it should be noted that for given series with length N and sampling period

$$T_s = \frac{1}{f_s}$$

where f_s is sampling frequency, total observation time is given by $N * T_s$, and spectral resolution limit is $\frac{1}{N*T_s}$, while concatenation of principal singular vectors allows us to increase resolving capacity, this method was tested on such situation, when the difference between two minimum frequency components was high than resolution limit of this signal, that means Δf was greater than $\frac{1}{N*T_s}$. In this article we consider the case of when this situation does not hold and will use an optimized and iterated SVD method to overcome this difficulty and to show that signal components can be separated by principal singular vectors even in the case of a difference between two minimum components is definitely smaller than $\frac{1}{N*T_s}$.

Methodology

The idea of iterated SVD method suggests the following: instead of concatenating principal singular vectors of Hankel matrix the first time and finishing the operation, we are continuing SVD decomposition on the new time series until we get much better picture than it was previously. We have the following matrix

$$X_d = \begin{vmatrix} x[1] & x[2] & \dots & x[p] \\ x[2] & x[3] & \dots & x[p+1] \\ \dots & \dots & \dots & \dots \\ x[N-p] & x[N-p+1] & \dots & x[N] \end{vmatrix}$$

where N samples of the time series $x[1], x[2],\dots,x[N]$ are given. It should be noted that main task of SSA (singular spectrum analysis) is to choose window length correctly (James B. Elsner, 1996), that means we should choose the window length that is enough to hold all principal components and at the same time it's singular vectors should be long enough to catch larger periodicities as much as possible, after applying SVD decomposition, which sorts singular values and their corresponding singular vectors, leading singular vectors contribute the main variance of the given signal and at the same time they repeat the same spectral structure of the original signal, but instead of using time series to concatenate these principal singular vectors for improving spectral resolution just once, we repeatedly apply SVD decomposition on the new time series. Basic idea of iterated SVD is that:

Idea of iterated SVD is that:

1. It can help us to increase signal to noise ratio more than it was at the beginning

2. It can increase data series much more in length, which again will have the same spectral structure as the original and therefore increases the spectral resolution

As the problem of not having sufficient data sample is so frequent in time series analysis, iterated SVD decomposition can give us comparatively better result than the traditional spectral estimation methods can do.

Experimental Results

Experiment 1

In the first experimental example we took the following data to analyze given signal, we have 4 deterministic components with frequencies

$f_1=13;\ f_2=12.6;\ f_3=12.4;\ f_4=13.02$;

With amplitudes $A_1=24\ \ A_2=A_3=A_4=23$;

We took sample size N=294 and sampling frequency f_s=100 Hertz, total observation time is 2.94 and frequency resolution limit is 0.34013605, we can easily see that first and last components are different from each other by less than 0.34013605 unit, also second and third components are also separated by less than this limit, spectral density of original series is

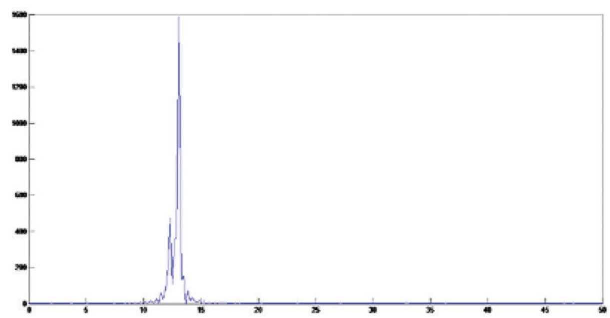

Figure 1. *Periodogram of original series*

From the picture it is clear because of not enough sample size, periodogram was not able to detect other peaks, we apply SVD with window size 35; picture of singular values is given below

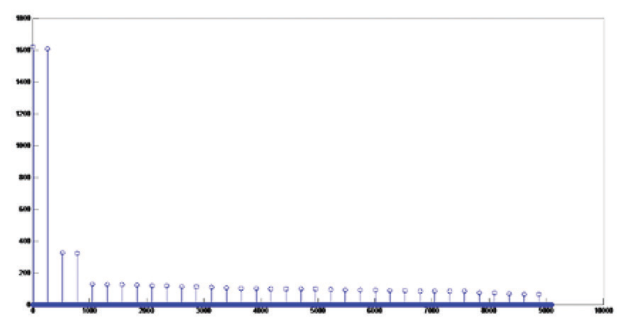

Figure 2. *Picture of singular values*

After analyzing each singular vector's spectral structure, we saw that first four singular vector have similar spectral structure as the original one, so we have concatenated

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for instance the first and the last singular vectors using matlab function vertcat, then again apply SVD decomposition of Hankel matrix created from this signal, the window length in this case is the same 35, from the new singular vectors we concatenated first and second and then obtain the following picture of power density from the concatenated signal using periodogram method

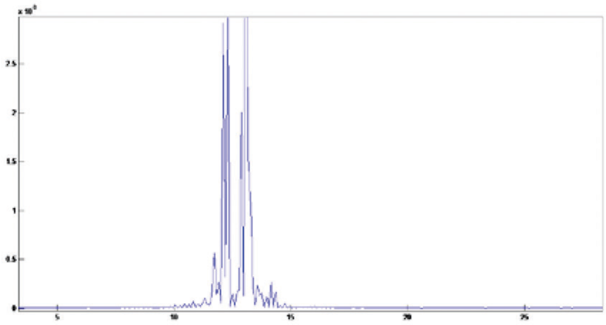

Figure 3. *Spectral structure of new time series*

If we zoom this picture in matlab we can see definitely the big 4 peaks comparately to its side lobes, these side lobes could be explained by complex structure of frequencies in the original signal, it should be mentioned that the length of the new series after the concatenation of the first two singular vectors in the second iteration is 972, while in the first case it was 294.

Experiment 2

In the second case we took a more complex case, like
f_1=18.001; f_2=18.003; f_3=18.002; f_4=18.004;
In this case we can easily see that they are too close to each other, so that its spectral density gives us just one peak, amplitudes are same, using periodogram we got following picture of original series

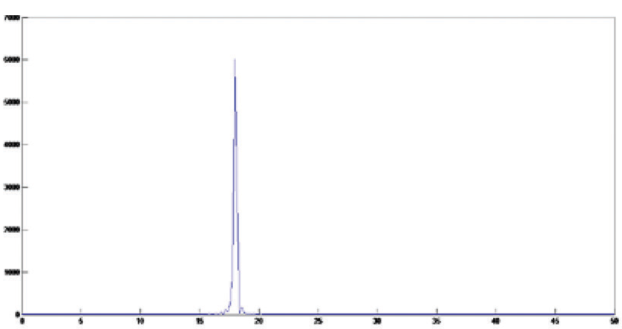

Figure 4. *PSD of original series*

In this experiment we now use 20 as the window size and get the following singular value plot . (Figure 5)

We can see that it is showing two leading singular values, which represents a complex conjugated pairs and is indicating that there exist only one deterministic component, while others are hidden in noise, we analyzed the spectral structure of corresponding singular vectors, and because they showed the same spectral structure, after concatenation, it was possible to discover another set of peaks as in the following picture. (Figure 6)

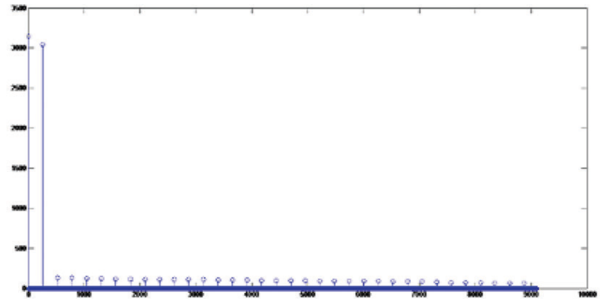

Figure 5. *Plot of singular vectors*

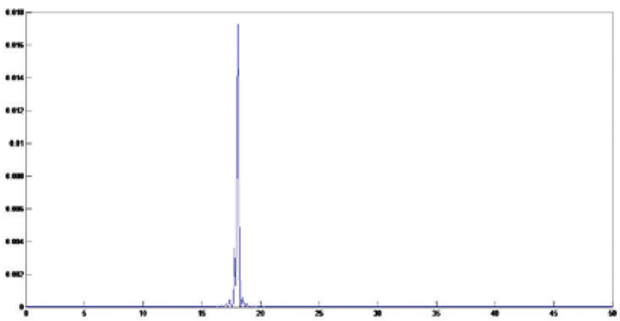

Figure 6. *PSD of concatenated singular vectors*

But still the other two peaks are lost, also the second peak is not clearly separated from the first, so we are going to apply SVD to this series, window length is the same 20, we concatenated the first two left singular vector and got following

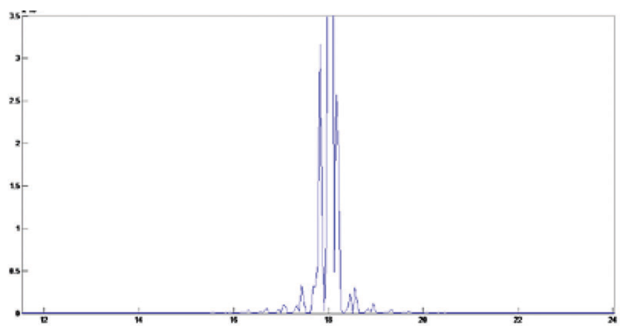

Figure 7. *Improved resolution of new series*

It should be noted that the length of this new series is 1062, while original was 294; if we continue this process, we get the picture in Figure 8.

If we zoom this picture, we can see that there are 4 peaks, but because the frequencies are so close to each other, that picture seems to be complex and need zooming, the length of the new series from which we get this spectral structure is 2086, while the previous was 294.

Result and Conclusion

As we see from the result, iterated SVD decomposition can be used additionally to original SVD to increase length of time series and also improve the spectral resolution, it was shown that even in case of non-enough sample data, we

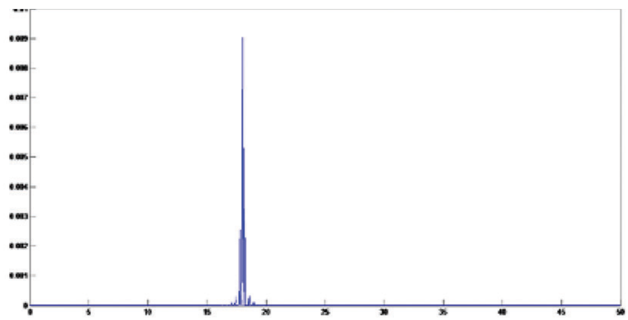

Figure 8

can increase the length of the sample data so that it is possible to detect frequencies that are so close even with less resolution. Of course the method's ability to improve spectral resolution may depend on the mathematical structure of the signal, amplitudes can be different from each other or there can be one dominant amplitude and the others are very small compared to the big one, but for close spaced amplitudes and frequencies, iterated SVD with the first SVD method can be used to get accurate picture and detect hidden periodicities, it should be noted for future development that the main question is how big is the role of window length of Trajectory matrix in optimization of the spectral structure, not only in iterated SVD method, but also in one SVD approach? Empirical or theoretical method for discovering suitable dimension of Hankel matrix that gives us an invaluable result to improve spectral resolution and stability of power spectral estimation for a given signal represented by periodic components.

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